

A.J. SADLER

MATHEMATICS
METHODS

UNIT
2



Mathematics Methods Unit 2
1st Edition
A. J. Sadler

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Preface.

This text targets Unit Two of the West Australian course *Mathematics Methods*, a course that is organised into four units, units one and two for year eleven and units three and four for year twelve.

UNIT ONE	UNIT TWO	UNIT THREE	UNIT FOUR
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The West Australian course, *Mathematics Methods*, is based on the Australian National Curriculum Senior Secondary course *Mathematical Methods*. Apart from small changes to wording, the unit twos of these courses are closely aligned but do differ in that the National Curriculum course, when considering differentiation, goes beyond polynomial functions and considers other linear combinations of power functions, and when considering antiderivatives, includes solving problems involving motion in a straight line. At the time of writing, these two aspects are not included in unit two of the West Australian course. I have found it appropriate to include the first of these items in this book, in chapter 5, and in later miscellaneous exercises, as it allows students to realise that the rule for differentiating ax^n , with respect to x , can be applied for n negative and/or fractional. To maintain alignment with National Curriculum I include the second item in chapter 8. West Australian users of this book can decide whether to include or omit these aspects.

In a few other places I have found it appropriate to go a little outside the confines of the syllabus for the unit. In this regard I include consideration of infinite geometric series, I include a few optimization questions involving functions that are not polynomials (as an extension exercise in chapter six) and on the basis that students are likely to encounter the integral sign on their calculators, I include this in the chapter on antidifferentiation.

The book contains text, examples and exercises containing many carefully graded questions. A student who studies the appropriate text and relevant examples should make good progress with the exercise that follows.

The book commences with a section entitled *Preliminary work*. This section briefly outlines work of particular relevance to this unit that students should either already have some familiarity with from the mathematics studied in earlier years, or for which the brief outline included in the section may be sufficient to bring the understanding of the concept up to the necessary level.

As students progress through the book they will encounter questions involving this preliminary work in the *Miscellaneous Exercises* that feature at the end of each chapter. These miscellaneous exercises also include questions involving work from preceding chapters to encourage the continual revision needed throughout the unit.

Some chapters commence with a "Situation" or two for students to consider, either individually or as a group. In this way students are encouraged to think and discuss a situation, which they are able to tackle using their existing knowledge, but which acts as a fore-runner and stimulus for the ideas that follow. Students should be encouraged to discuss their solutions and answers to these situations and perhaps to present their method of solution to others. For this reason answers to these situations are generally not included in the book.

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Important note.

This series of texts has been written based on my interpretation of the appropriate Mathematics Methods syllabus documents as they stand at the time of writing. It is likely that as time progresses some points of interpretation will become clarified and perhaps even some changes could be made to the original syllabus. I urge teachers of the Mathematics Methods course, and students following the course, to check with the appropriate curriculum authority to make themselves aware of the latest version of the syllabus current at the time they are studying the course.

Alan Sadler

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